QUIZ 18 SOLUTIONS: LESSON 25 NOVEMBER 1, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [10 pts] Find the extrema of x^3y^2 subject to x = 1 + y.

Solution: We have $f(x,y) = x^3y^2$ and g(x,y) = x - y = 1. By the method of Lagrange multipliers, this means that

$$3x^{2}y^{2} = \lambda$$
$$2x^{3}y = -\lambda$$
$$x - y = 1$$

The first two equations imply

$$3x^2y^2 = -2x^3y.$$

We break this into cases.

Case 1: x = 0

By our constraint, this implies $-y = 1 \Rightarrow y = -1$. Thus, one solution is (0, -1).

Case 2: y = 0

By our constraint, we see x = 1. Hence, another solution is (1, 0).

Case 3: $x \neq 0, y \neq 0$

In this case, we can divide by both x and y. Dividing

$$3x^2y^2 = -2x^3y$$

on both sides by x^2y , we get

$$3y = -2x \Rightarrow y = -\frac{2}{3}x.$$

By our constraint,

$$x - \left(-\frac{2}{3}x\right) = 1 \Rightarrow \frac{5}{3}x = 1 \Rightarrow x = \frac{3}{5}.$$

Then,

$$y = -\frac{2}{3} \left(\frac{3}{5}\right) = -\frac{2}{5}.$$

Now, we check our function values at these points:

$$f(0,-1) = (0)^3(-1)^2 = 0$$

$$f(1,0) = (1)^3(0)^2 = 0$$

$$f\left(\frac{3}{5}, -\frac{2}{5}\right) = \left(\frac{3}{5}\right)^3 \left(-\frac{2}{5}\right)^2 = \frac{27(4)}{5^5} = \frac{108}{3125}.$$

Thus, the minima is $\boxed{0}$ and the maxima is $\boxed{\frac{108}{3125}}$